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CENTRALIZED REFUGEE MATCHING MECHANISMS WITH HIERARCHICAL PRIORITY CLASSES

Dilek Sayedahmed

WISIR, University of Waterloo, Canada
dilek.sayedahmed@uwaterloo.ca

ABSTRACT

This study examines the refugee reallocation problem by modeling it as a two-sided matching problem between countries and refugees. Based on forced hierarchical priority classes, I study two interesting refugee matching algorithms to match refugees with countries. Axioms for fairness measures in resource allocation are presented by considering the stability and fairness properties of the matching algorithms. Two profiles are explicitly modeled—country preferences and forced prioritization of refugee families by host countries. This approach shows that the difference between the profiles creates blocking pairs of countries and refugee families owing to the forced hierarchical priority classes. Since the forced priorities for countries can cause certain refugees to linger in a lower priority class in every country, this study highlights the importance of considering refugees' preferences. It also suggests that a hierarchical priority class-based approach without category-specific quotas can increase countries' willingness to solve the refugee reallocation problem.

Keywords: Stability, deferred acceptance algorithm, refugee studies.

JEL Classification Numbers: D47, C78, D78, I30.

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1. INTRODUCTION

IN the international refugee regime, it has posed a real challenge to finding a solution to the European refugee crisis. European countries have been reluctant to participate in the responsibility-sharing of resettling refugee families. Given this context, this study investigates the problem of resource allocation. Furthermore, this study draws inspiration from Abdulkadiroğlu & Sönmez (2003), in which they formulate the school choice problem as a mechanism design problem. They propose two competing mechanisms—the student optimal stable mechanism (Gale & Shapley, 1962) and the top trading cycles mechanism—each providing a solution to the school choice problem; see Shapley & Scarf (1974), Pápai (2000), and Abdulkadiroğlu & Sönmez (2003) on the top trading cycle mechanism. In this study, we reformulate the refugee reallocation problem as a mechanism design problem to address to the critical international refugee management issues. I define a two-sided matching problem with refugee and country preferences as a *country acceptance problem*. The one-sided version of this matching problem would be with refugees' preferences and countries' predetermined priority rankings. Consequently, based on the mechanism design literature, these problems are similar to college admissions and school choice problems, respectively.

Here two questions arise naturally. First, how can we model the interplay between forced priority classes and country preferences, that is, how can we capture the conflict between a one-sided and a two-sided model in the refugee reallocation context? Although the preferences of countries seem to be frowned upon in political debates, the global reality indicates that for a stable responsibility-sharing, preferences of countries tend to be crucial and influential. Second, what type of stability measures could be outlined for such a problem? I address the country acceptance problem by designing two matching algorithms based on forced hierarchical priority classes. These algorithms could be implemented as centralized refugee matching systems that match refugee families to countries. In this study, the term "refugee" is used in reference to a refugee family. In designing a centralized matching system that ensures the inseparability of refugee families that do not wish to be separated, this study assumes the implementation of a clearinghouse to accept preference submissions from households, namely, refugee families. All participating countries in the clearinghouse treat refugee households as a single refugee family unit.

Moreover, I conduct an axiomatic allocation analysis by focusing on the stability and fairness properties of the matching mechanisms. I explicitly model and analyze two profiles—countries' preferences, and the prioritization of refugee families imposed on host countries. Having two kinds of ranking profiles for countries, a *forced priority profile* and a *preference profile*, allows us to capture the difference between the two profiles. This, thereby, creates blocking pairs of countries and refugees owing to the forced hierarchical priority classes. The forced priority profile is a joint master list that is designed according to the United Nations High Commissioner for Refugees' (UNHCR) principles. It is applied to all countries, which may conflict with some countries' preferences. Similar models have been used in the resident allocation problem with distributional constraints in the computer science and artificial intelligence literature; see e.g., Goto et al. (2016).

In this context, I recognize the importance of investigating the weakening of the stability and fairness axioms. A stable mechanism may no longer satisfy the standard stability of the literature concerning country preferences, leading to potential blocking pairs. Therefore, I contribute to the literature by studying weaker stability and fairness axioms in order to determine the type of stability and fairness properties that hold in this setting. The motivation behind this is twofold. First, I account for the fact that countries have their preferences. Second, the UNHCR has strict humanitarian guidelines and principles for refugee settlement, which are a pivotal consideration when designing a centralized refugee matching mechanism (Assembly et al., 1951; Szobolits & Sunjic, 2007). Based on these guidelines, I impose priority classes on participating countries that force countries to change their preference rankings. This, however, leads to a deviation from the goal of a fair refugee allocation. Owing to these forced priorities for the countries giving certain refugees a priority in *each* country, I recognize the need for and the importance of considering refugee preferences. Since priority classes are forced in all countries, every refugee in the first priority class (PC) is always prioritized over others. Thus, in my proposed system, a refugee in a lower category remains in that category. Please note that the abbreviation "PC" is used for "priority class" throughout this study.

Considering the refugees who linger in the lower forced priority classes, I focus on two different forms of priority profiles to give these refugees an additional chance to improve their ranking. In my first form, namely the top prioritization mechanism, I provide these refugees with a higher probability to be matched with their top-ranked country. In my second form, I present them with a better chance of being paired with their Deferred-Acceptance-matched country. It must be noted that, throughout this study, I refer to Gale & Shapley (1962)'s refugee-proposing Deferred-Acceptance mechanism as "the DA," which is applied to the refugee and country preferences. The second form is defined as the DA-match prioritization mechanism. Moreover, unlike the first form, I find that the DA-match is the best matching that the refugees in lower priority classes can acquire in a stable matching scenario. This shows the importance of prioritizing these refugees in their DA-matched countries.

For countries, the prioritization of these refugees in their DA-matched countries is more compelling, given that each country's priorities undergo fewer quota-based modifications than those in the first form of a priority profile based on refugees' top-ranked countries. Under the first form of a priority profile, it may not be necessary for some countries to modify the priority orders, despite it being mandatory for others to do so, and to do so to a greater extent. Let us consider Germany—the desired country for refugee settlement. When Germany becomes the top choice for a large number of refugee families, the country will move several refugees to its top priority class, ranking them according to its own point system. This system represents Germany's preferences. Consequently, a favored country such as Germany can make several changes to its priority ordering. This will cause the country to deviate significantly from the forced priority compared with a less popular country among refugees.

This study's contribution lies at the intersection of the matching theory and refugee studies through multiple channels. First, although countries may have clear preferences for refugees, they are not required to be familiar with all the predispositions over the entire set of refugees to run the mechanisms designed in this study. Since these mechanisms would require the countries to submit preferences over refugees in the same priority class, it would enable the countries to implement the mechanisms more efficiently. Second, challenges may arise from the imposition of type-specific (e.g., PC-specific), set-aside reserve quotas on countries in a refugee allocation setting. Hence it may not be well-accepted by the countries. However, a hierarchical priority class-based approach without category-specific set-aside reserve quotas may be more acceptable and induce more countries to willingly solve the refugee reallocation problem. This approach would persuade more countries to participate in a centralized refugee matching mechanism. This study's findings have other important, decisive policy implications. For example, they can be applied to centralized college admissions, the design of public-school choice systems, and an immigration process characterized by a more effective priority-based than a category-specific reserve quota system.

Literature Review

Top prioritization mechanism may seem part of the first choice maximizing (FCM) mechanisms of Dur et al. (2018). The motivation of their study of FCM mechanisms is the common focus on first choices in school choice markets and the popularity of variants of the Boston mechanism (BM) in practice. BM is popular and it has an intuitive way of attempting to maximize first choices. However, BM was not part of the motivation of this study. The top prioritization mechanism was motivated by the need to identify the means of helping refugees who face the risk of being stuck in a low priority class in all countries. When searching for a way to help move these refugees up to a higher priority class, a natural starting point was to look at these refugees' most-preferred countries. BM can be understood as DA with a modified priority profile, where one first sorts agents according to their rank of the object (agents who rank the object first are in the top PC, agents who rank the object second are in the second PC, etc.) and, within each PC, the agents are ordered according to the original priorities of the object.

In contrast, the top prioritization mechanism is simply the DA applied to a newly adjusted forced priority profile in which refugees are moved up to top PC of their most-preferred countries. Moreover, when these refugees are promoted to top PC, they are still ranked according to country preferences within the top PC. Meanwhile, the first choices first (FCF)-algorithm of Dur et al. (2018) is a procedure in which at step one each student applies to her respective first choice school; and each school accepts applicants into open seats according to priorities until there are no more applicants or all seats are filled. In step two, rejected students are matched to open seats by an arbitrary procedure but without changing the matchings that were made in step one. Furthermore, the top stability axiom of this study, which is a weakened stability axiom that allows for blocking pairs of refugees and countries that are not their top choice, is Dur et al. (2018)'s first choice-stability. A matching is first choice-stable if no student forms a blocking pair with her first choice.

Credible stability is one of the other weak stability axioms examined in this study. It allows for salient blocking pairs that are not matched under the DA in the refugee allocation setting. It is an interesting concept as it can potentially improve upon the DA refugee-optimal solution, although without being strategyproof. This axiom may remind the reader of the Optimal Priority DA proposed in Biró & Gudmundsson (2021). In their study, they examine the complexity of finding Pareto-efficient allocations of highest welfare in a school choice context. The Optimal Priority DA is a DA executed on the instance with priorities adjusted to the welfare-maximizing allocation based on minimized distance. The idea of the Optimal Priority DA is that the authors take an optimal matching (e.g. one that minimises the total travel distance for the students) and they provide top priority to all students at the schools where they are assigned in this socially optimal solution. Then they apply a standard DA taking into account the students' preferences and the further priorities at the schools. While Optimal Priority DA starts by computing the welfare-maximizing allocation, the DA-match prioritization mechanism depends on the initial DA allocations based on country and refugee preferences. Moreover, since the DA-match prioritization mechanism weakly improves on the DA and is manipulable, domains studied in Kesten & Kurino (2019) are of interest to the readers. They identify maximal domains on which strategy-proof mechanisms dominating DA exist. The motivation of their paper is different than this study, as their main goal is the improvement of the DA and examining the level of strategy-proofness loss as a result. Meanwhile, in this study, the DA-match prioritization mechanism is designed as an alternative channel to give refugees, who face the risk of being stuck in a lower PC, an improved rank by moving them up to a higher PC. The Pareto-improvement result that came with the DA-match prioritization mechanism's design was a pleasant surprise.

Since the two mechanisms of top and DA-match prioritization in this study are manipulable, they are of interest with respect to the mechanisms studied in Pathak & Sönmez (2013). Their approach to studying a mechanism's vulnerability to manipulation is to characterize domains under which the mechanism is not manipulable. They develop a rigorous methodology to compare mechanisms based on their vulnerability to manipulation. Moreover, the weak stability of Pathak & Sönmez (2013) seems similar to the top stability of this study. Their weak stability is a relaxation of stability; for example, students are allowed to block matchings only with their top choice schools. Meanwhile, under the top stability of this study, which is also a relaxation of stability, such blocking pairs are the ones that are not allowed.

The remainder of this paper proceeds as follows. Section 2 provides a background on the refugee reallocation crisis, elaborates on this study's motivation, and outlines the centralized system proposed along with the UNHCR-mandated priority classes. Section 3 presents the model. Section 4 investigates how we can weaken stability in the context of hierarchical UNHCR-mandated priority classes. This section also provides the basic definitions of the axioms and related theorems. Sections 5 and 6 present the two weak stability axioms that consider the top-ranked and DA-matched countries when modifying priorities that improve the chance of refugees in the lower priority classes. Section 7 concludes this study.

2. BACKGROUND AND MOTIVATION

The number of global refugees is currently at its highest level since the Second World War, and Europe has attracted the predominant mass of refugees (Alfred, 2015). This scenario shows the European Union's (EU's) inconsistency and poorly coordinated response to refugees fleeing the collapse of Syria. This issue has attracted several studies seeking solutions to the refugee allocation problems at the international and local levels. Considering the local refugee match within a country, studies have presented several obstacles to successful refugee integration. Andersson & Ehlers (2020) focus on the problem of finding housing for refugees after their resettlement to an EU country. They propose an easily implementable algorithm for Sweden that finds a stable maximum matching. Further, Bansak et al. (2018) and Delacrétaz et al. (2019) focus on different aspects of the refugee allocation problem: the former on the optimization of refugee preferences and the latter on family size.

In the context of the international refugee allocation problem, there have been several calls for revising or replacing the Dublin Regulation, particularly its requirement that the first EU country that gives asylum undertake the responsibility of processing the asylum seekers' claims (Giuffre & Costello, 2015; Koser, 2011). The current decentralized system, which assigns this responsibility to the first-arrival country, has been unfair to border countries such as Greece, Italy, or Hungary. Subsequently, this system has also been responsible for creating chaos and tragedy. One article criticizes the European states for playing "pass-the-parcel with human lives (Jones & Teytelboym, 2017)." I agree with Jones & Teytelboym (2017) that "it has never been clearer that a new deal on responsibility-sharing within Europe is needed to replace the Dublin Regulation."

It is crucial to implement a centralized matching system alongside or, ideally, instead of the current system of the Dublin Regulation. A new centralized system would allow a refugee family to apply for protection in more than just one country, thus not binding them to a single application to a particular country. Under the current system, refugees take the gamble of deciding where to apply, and countries cannot evaluate and choose from a large pool of applicants.

A centralized mechanism would allow all refugees to apply to a single system from any embassy, and this could benefit both refugees and countries (Jones & Teytelboym, 2017, 2016). Just as a centralized mechanism could help refugees be better off by helping them avoid dangerous cross-border journeys, a centralized mechanism could also enable countries to gain more control than they currently possess in deciding who settles within their borders. This can be ensured by allowing countries to assign their preference ranking on which refugees they wish to accept, similar to refugees citing their preferences for countries (Jones & Teytelboym, 2017, 2016).

Furthermore, a centralized matching system would allow refugees to apply for protection in several countries while allowing the countries to compete for refugees. This system would require the refugees to make only a single claim for asylum to a single centralized body while simultaneously specifying their country preference. When the countries approach the clearing house with a quota and ranking of refugees, they are willing to accept that the system can be implemented to match the refugees to the countries. After this matching process, it would be crucial to implement this match, that is, refugees are granted refugee status and permitted to settle in the country to which they have been matched. A centralized system would allow refugees to apply for asylum with *every* participating country, and they can, in principle, submit it remotely. This submission would include the regional processing centers in the Middle East and North Africa (Jones & Teytelboym, 2017, 2016). The core advantage of the system is that it provides refugee families with confidence that a fair and effective system will grant them protection, and they will be less likely to take the risk of a dangerous crossing.

Moreover, the UNHCR's *Convention and protocol relating to the status of refugees* (Assembly et al., 1951) is both a status- and rights-based instrument. It is underpinned by several fundamental principles, such as safety and protection, non-discrimination, non-penalization, and *non-refoulement*. Convention provisions, for example, are to be applied without discrimination as to race, religion, or country of origin. Developments in international human rights law also reinforce the principle that the Convention be applied without discrimination based on sex, age, disability, sexuality, or other prohibited grounds of discrimination (Assembly et al., 1951).

The UNHCR document states the following:[1]

"The conference, considering that the unity of the family—the natural and fundamental group unit of society—is an essential right of the refugee, and that such unity is constantly threatened, and noting with satisfaction that, according to the official commentary of the *ad hoc* committee on 'Statelessness and Related Problems,' the rights granted to a refugee are extended to members of his family, recommends governments to take the necessary measures for the security and protection of the refugee's family, especially with a view to:

1. Ensuring that the unity of the refugee's family is maintained particularly, in cases where the head of the family that has been waiting for admission has fulfilled the necessary conditions for admission to a particular country,

2. The protection of refugees who are minors, in particular unaccompanied children and girls, with special reference to guardianship and adoption (Assembly et al., 1951)."

The Forced Priority Class Hierarchy: A Proposal

A centralized system aims to allocate refugees within that system to a country where they are most likely to flourish during their time of residency, without causing further immigration spillover to other countries (Jones & Teytelboym, 2017, 2016). It is essential to ensure that such a system excludes discriminatory categories and focuses on the categories of vulnerability, the suitability for integration, and the presence of family. Hence, I propose *forced hierarchical priority classes* of refugees for countries based on the UNHCR's 1951

[1] For the full texts of the UNHCR 1951 documents, please see the Convention and Protocol relating to the status of refugees from the UN General Assembly in Geneva (Assembly et al., 1951).

convention and protocol for refugees (Assembly et al., 1951). Throughout this study, I refer to this UNHCR-mandated PC hierarchy of refugees as the *forced priority classes*. A forced PC of refugees is a subset of the entire set of refugees a country must consider as part of the PC hierarchy, which is the same for all participating countries. The forced PC hierarchy is imposed exogenously as part of the centralized system. The proposed forced priority profile for countries is then as follows:

- *Priority Class I:* Refugee families in war zones and with the longest waiting period
- *Priority Class II:* Refugee families in war zones
- *Priority Class III:* Refugee families with the longest waiting period
- *Priority Class IV:* Other refugee families

Given the initial theoretical approaches to complicated real-life problems, such as the refugee crisis, it is a requisite to start the analysis with a static model. Therefore, for its theoretical tractability, this study chose to take a static approach. The key motivations for this study are the Syrian refugee crisis and the reallocation problems resulting from the Syrian war. The Syrian crisis has led to an influx of refugees into other countries, which creates a refugee pool. Thus, we can conduct this exercise repeatedly. However, in the school choice, there are natural time periods. For example, the implementation of school choice would require considering each academic year. However, it is unclear whether the same process can be applied to refugee settlement with a sudden influx of refugees. Nonetheless, since this exercise can be implemented, for instance, every three months when there is a crisis, we can consider a dynamic approach. By heeding this approach, a static approach would also be crucial to address a refugee crisis that creates a sudden large pool of refugee families requiring immediate allocation.

Usually, dynamic models can better capture the essence of the problem under study than static models do. In dynamic models, agents nn arrive at different time periods and, sometimes, continuously; matching takes place at different time periods, and the matched agents leave the market, and new agents arrive. When this is the case, we lose some desirable properties in a static matching. Thus, we can turn the matching environment of refugee allocation into a static environment and retain desirable properties of stability and efficiency. When possible, it would be ideal to array agents from both sides to conduct repeated static matchings. This would be an optimal approach in an environment with a crisis and is better than the dynamic approach of performing repeated matchings in each time period as refugees enter and leave the market. Hence, in the case of a sudden influx of refugee families, the static approach can be adapted to facilitate matching these participants, who are fixed at the moment of the crisis when the matching takes place.

This study contributes to the literature as the model explicitly allows for the potential real-world "discrepancy" between what countries actually want to do (i.e., country preferences) and what countries are forced to do by law (i.e., countries' forced priorities). By working with two profiles for countries, I capture the "compromise" between countries' *actual* preferential ranking of refugees in a world with no special considerations and their *forced* priority hierarchy based on the UNHCR's humanitarian laws and principles. Forced priorities are used for school choice by forcing the local authorities to use the main priority categories, such as living in the catchment area and/or having a sibling in the school. This study introduces the perspective of compromise between desired and forced preferences to the refugee reallocation problem by making it the central topic of the policy-making discussion. Moreover, this study explicitly lays out this compromise and its potential implications for fairness. In the refugee reallocation context, country preferences are negatively reviewed due to discrimination issues. Therefore, the hierarchical classes would aid in cutting down significantly on this kind of discrimination in the refugee setting by imposing the UNHCR-based priority classes, which makes country preferences much less important for the matching outcome. However, on the other hand, refugees face the risk of being stuck in lower priority classes in all countries. This study contributes by attempting to find a middle ground between these two forces by designing mechanisms that help those refugees who might face the risk of remaining in lower priority classes.

3. THE MODEL

Definition (Country Acceptance Problem). A problem consists of the following:

1. A finite set of refugees $\mathscr{R} = \{r_1, r_2, ..., r_{|\mathscr{R}|}\}$.

2. A finite set of countries $\mathscr{C} = \{c_1, c_2, ..., c_{|\mathscr{C}|}\}$.

3. A quota vector $q = (q_{c_1}, ..., q_{c_{|\mathscr{C}|}})$, where q_c is the capacity (the number of residency permits) of $c \in \mathscr{C}$.

4. Preference profile of refugees $P = (P_{r_1}, P_{r_2}, ..., P_{r_{|\mathscr{R}|}}) = (P_r)_{r\in\mathscr{R}}$, where P_r is the strict preference of refugee $r \in \mathscr{R}$ over $\mathscr{C}$.

5. Preference profile of countries $\succ = (\succ_{c_1}, \succ_{c_2}, ..., \succ_{c_{|\mathscr{C}|}}) = (\succ_c)_{c\in\mathscr{C}}$, where $\succ_c$ is the strict preference of country $c \in \mathscr{C}$ over $\mathscr{R}$, based on its country-specific point system.[2]

A finite set of refugees $\mathscr{R}$ is with a *fixed partition*, such that $\mathscr{R} = \mathscr{R}_1 \cup \mathscr{R}_2 \cup ... \cup \mathscr{R}_T$ and $\mathscr{R}_t \cap \mathscr{R}_{t'} = \emptyset$, for any $t, t' \in \{1, ..., T\}$). The T number of *forced hierarchical priority classes* of the partition are class $\mathscr{R}_1$, class $\mathscr{R}_2$, and so on until class $\mathscr{R}_T$. These are the forced priority classes introduced and discussed in the previous section.
As a primitive version of the model, I also define the following:

Definition (Forced Priorities). Countries' enforcing priority profile $\pi^E = (\pi^E_c)_{c\in\mathscr{C}}$ is based on the fixed partition $\mathscr{R}_1, ..., \mathscr{R}_T$ of the refugees into the forced priority classes.

Let $t, t' \in \{1, ..., T\}$ such that $t \leq t'$ and let $r_i \in \mathscr{R}_t$, $r_j \in \mathscr{R}_{t'}$. Then, for all $c \in \mathscr{C}$,

a. If $t = t'$, then $r_i \; \pi^E_c \; r_j$ if and only if $r_i \succ_c r_j$.

b. If $t < t'$, then $r_i \; \pi^E_c \; r_j$.

For item (a) above, observe that each country's rankings within each PC are country-specific preferences. For any $c \in \mathscr{C}$ with $r_i \succ_c r_j$, if r_i and r_j are prioritized within the *same* class, then *within* that PC, country c's ranking will also be $r_i \; \pi^E_c \; r_j$, following country c's preference ranking. For item (b) above, observe that if one refugee is in a higher PC than another refugee, then this implies that, according to π^E, the refugee in the higher priority class will be

[2] Unacceptable countries are not allowed in the model, in which case each refugee family $r \in \mathscr{R}$ has strict preferences over $\mathscr{C}$, where a country c is never ranked below r. Similarly, unacceptable refugees are not considered in the model. Hence, each country $c \in \mathscr{C}$ has strict preferences over $\mathscr{R}$, where a refugee r is never ranked below c.

strictly prioritized over the refugee who is in a lower PC.

Remark. In partitioned forced priorities, each member of the partition corresponds to an exogenously imposed PC. The priority classes are forced across all participating countries; each forced PC consists of the same set of refugees for all participating countries.

In summary, a many-to-one *country acceptance problem*, where each refugee family can be matched to a maximum of one host country and each country can admit a maximum of q_c refugee families, is defined as the tuple

$$\langle \mathscr{R} = \bigcup_{t=1}^{T} \mathscr{R}_t, \mathscr{C}, (q_c)_{c \in \mathscr{C}}, P, \succ \rangle.$$

When all other parameters, except that of the refugees' preference profile P, are fixed, I refer to P as a country acceptance problem. Let $\mathscr{P}$ denote the set of country acceptance problems.

To simplify the exposition, I assume that for all $c \in \mathscr{C}$, $|\mathscr{R}| > q_c$. This assumption is easily satisfied in any application and allows the rejection of the case where there are few refugees. I also perform the following notation: Let W_r denote the weak preferences of refugee $r \in \mathscr{R}$ associated with P_r.[3] Since preferences are strict, $c \; W_r \; c'$ means that either $c \; P_r \; c'$ or $c = c'$. The preferences of a coalition $L \subseteq \mathscr{R}$ in P are denoted by P_L. Finally, I denote the preference profile of all the refugees, except for r by P_{-r}, and the preference profile of all refugees, except the ones in coalition L by P_{-L}.

For simplicity, I assume that countries have *responsive preferences* over refugee families. This means that relatively speaking, refugee families are not complements in the countries' preferences. Thus, preferences over sets of refugee families can be interpreted as a natural extension of preferences over individual refugees.

Definition (Responsive Preferences). For any set of refugee families, $\mathscr{Z} \subset \mathscr{R}$ with $|\mathscr{Z}| \leq q_c$ and any refugee family r and r' in $\mathscr{R} \setminus \mathscr{Z}$,

- $\mathscr{Z} \cup \{r\} \succ_c \mathscr{Z} \cup \{r'\}$ if and only if $r \succ_c r'$.

[3] Please note that symbol W is used to denote weak preference relation for refugees. This is done to avoid confusion between the common notation that is used for weak preference (R) associated with P and set of refugees $\mathscr{R}$.

It must be noted that the notation has been slightly abused in the above definition. To indicate preferences over sets, $\succ_c$ is used; it is normally used for showing preferences over singletons in $\mathscr{R}$. Responsive preferences are a natural extension of preferences over individuals to preferences over sets. This property does not give a complete ordering of all the sets of size q_c for a country, as it does not determine all the preference rankings over sets. However, this does not affect this study's analysis. It is not necessary to determine the missing preference orderings over sets, and they can be in any order.

Definition (Matching). A solution to a many-to-one refugee matching problem is μ: $\mathscr{R}\bigcup\mathscr{C} \to \mathscr{R}\bigcup\mathscr{C}$, a correspondence from $\mathscr{R}\bigcup\mathscr{C}$ to $\mathscr{R}\bigcup\mathscr{C}$ such that, for every refugee family $r \in \mathscr{R}$ and participating country $c \in \mathscr{C}$:

- $\mu(r) \in \mathscr{C}$
- $\mu(c) \subseteq \mathscr{R}$ and $|\mu(c)| \leqslant q_c$
- $\mu(r) = c \Leftrightarrow r \in \mu(c)$

Let μ and ν be two matchings. A matching μ *Pareto dominates* matching ν at preference profile $P \in \mathscr{P}$ if for all refugees $r \in \mathscr{R}$, $\mu_r W_r \nu_r$, and there exists a refugee r such that $\mu_r P_r \nu_r$. A matching μ *weakly Pareto dominates* a matching ν at P if either μ Pareto dominates ν or μ is the same as ν.
The set of problems is denoted with $\mathscr{P}$, which is

$$\langle \mathscr{R} = \bigcup_{t=1}^{T} \mathscr{R}_t, \mathscr{C}, (q_c)_{c\in\mathscr{C}}, P, \succ \rangle.$$

Let $\mathscr{M}$ be the set of matchings. Since everything else, except P is fixed, I define a mechanism as follows.

Definition (Mechanism). A mechanism is a mapping that assigns a matching to each country acceptance problem $P \in \mathscr{P}$. Formally, a mechanism is a mapping $f : P \to \mathscr{M}$.

Let f and g be two mechanisms. A *mechanism f Pareto dominates mechanism g* for $\mathscr{R}' \subseteq \mathscr{R}$ if for all profiles $P \in \mathscr{P}$, $f_{\mathscr{R}'}(P)$ weakly Pareto dominates $g_{\mathscr{R}'}(P)$, and there exists a $\bar{P} \in \mathscr{P}$ such that $f_{\mathscr{R}'}(\bar{P}) \neq g_{\mathscr{R}'}(\bar{P})$. If *mechanism f weakly Pareto dominates mechanism g*, then, at every P, mechanism f produces a matching that weakly Pareto dominates the matching produced by g.

Since this study focuses on the refugee-proposing DA and its modifications, the matching results differ with respect to the type of country profile used. A refugee assignment mechanism requires refugees to submit preferences for countries and selects a matching based on these preferences and refugee priorities. Note that the notation μ^{DA} is used for matching results obtained from the refugee-proposing DA applied to P and $\succ$.

Definition (Blocking Pairs). A matching μ is blocked by a refugee–country pair $(r,c) \in \mathscr{R} \times \mathscr{C}$ if they prefer each other, relative to μ:

1. the refugee family r prefers c to the country to which it is matched μ (i.e., $c \; P_r \; \mu(r)$), and

2. given $r \notin \mu(c)$,

    a. either the country prefers r to some refugee r' that the country is matched to in μ (i.e., $r \succ_c r'$ where $r' \in \mu(c)$)

    b. or refugee r is acceptable to country c and the country has fewer refugee families assigned to it than its quota (i.e., $r \succ_c c$ and $|\mu(c)| < q_c$).

Regarding the definitions, recall that this study uses the term "refugee" when referring to a "refugee family." We now formally define stability using the notion of blocking pairs, which will be important for the axioms studied throughout this study.

Definition (Stability). A matching is stable if it is not blocked by a refugee–country pair. A mechanism is stable if it assigns a stable matching to each country acceptance problem P.

We now adapt the well-known algorithm of Gale & Shapley (1962) to our current model and call it the *Refugee-Proposing Deferred Acceptance (DA) Mechanism*.

Step 1: Each refugee proposes their first choice country. Each country tentatively assigns its refugee residency permits to its proposers based on its quota, following its preference order. Subsequently, any remaining proposers are rejected.

In general, at

Step k: Each rejected refugee, in the previous step, proposes the succeeding country of choice. Each country considers the refugees it has been holding along with its new proposers and tentatively assigns its refugee residency permits up to its quota, following its preference order. Subsequently, any remaining proposers are rejected.

The mechanism terminates when no refugee proposal is rejected, and each refugee is assigned the final tentative assignment. We refer to this mechanism as the *Gale-Shapley refugee-optimal stable mechanism*.

Gale & Shapley (1962) call this stable mechanism *optimal* if every refugee is at least as well off under it as under any other stable matching. Furthermore, the DA procedure yields not only a stable matching but also an optimal one (Gale & Shapley, 1962). Every refugee is at least as well off under the matching assigned by the DA mechanism as they would be under any other stable matching. This holds for both sides—refugees and countries. For every country acceptance problem, there exists a refugee-optimal stable matching, which is at least as agreeable to each refugee as any other stable matching. There also exists a country-optimal stable matching, which is at least as agreeable to each country as any other stable matching. The refugee-proposing DA mechanism leads to the refugee-optimal stable mechanism. Throughout this study, DA's matching result is denoted by μ^{DA}.

4. WEAK STABILITY

When applying a mechanism using the forced priority class profile π^E there may exist blocking pairs due to how refugees are ranked according to country preferences $\succ$. The discrepancy between refugees' ranks in π^E and $\succ$ may result in a stable mechanism with respect to π^E being no longer stable with respect to $\succ$. Therefore, when a refugee family forms a blocking pair with a country, this blocking will always be with respect to another refugee family in a higher forced PC rather than the refugee family forming a blocking pair. The basis for blocking is the priority reversal resulting from the forced priority classes. This observation inspires the following axioms, the first of which is my definition of the first weak fairness axiom of this study.

Definition (PC Fairness Axiom). A matching μ satisfies PC fairness at a particular profile P if, for every refugee–country pair (r,c) such that r prefers c to own assignment at μ and is preferred by c to a refugee $\hat{r}$ assigned to c at μ, $\hat{r}$ is in a higher PC than r. A mechanism f satisfies PC fairness if, for every profile P, it assigns a matching μ that is PC fair.

According to PC fairness, if a refugee–country blocking pair (r,c) is such that r is preferred by c to a refugee $\hat{r}$ assigned to c, then it must be the case that $\hat{r}$ is in a higher PC than r. In order to discuss the intuition behind the axiom, we can say that the PC fairness axiom tracks the discrepancy between forced priorities and country preferences, and it allows for certain salient blocking pairs accordingly. Since we are weakening stability to respect the urgency of placing refugees who are prioritized in higher classes, stability implies PC fairness. PC fairness is an axiom that requires stability within each PC of a given hierarchical structure, based on country preferences that are preserved within each PC.

Definition (PC No-Envy Axiom). A matching μ satisfies PC no-envy at a particular profile P if $\hat{r}$ is in a higher forced priority class than r and $r \in \mu(c)$, then $\mu(\hat{r})\ W_{\hat{r}}\ c$. A mechanism f satisfies PC no-envy if, for every profile P, it assigns a matching μ that satisfies PC no-envy.

A case in point: For each preference profile P, given the matching μ assigned to P where r is among the refugees matched to c, if another refugee $\hat{r}$ is in a higher forced priority class than r, then $\hat{r}$ does not have envy for r.

To demonstrate the independence of the PC no-envy and PC fairness axioms, observe that, given a fixed profile P, the matching result of the DA, specifically μ^{DA}, is fair and thus satisfies PC fairness. However, μ^{DA} does not satisfy PC no-envy, as it is not stable with respect to the forced priority classes of π^E at each profile P. Hence, PC fairness does *not* imply PC no-envy. Consider the following example to intuitively visualize this case:

Example 1. Let us consider the problem P with refugees $\mathscr{R} = \{1,2,3,4\}$, countries $\mathscr{C} = \{a,b,c,d\}$, country quotas $q_c = 1$ for all $c \in \mathscr{C}$, and forced priority classes $\mathscr{R}_1 = \{1,2\}$ and $\mathscr{R}_2 = \{3,4\}$. Allocations of the matching result μ^{DA} at the given preference profile, in this example, are in *bold*.

P_1	P_2	P_3	P_4
c	b	c	b
b	c	b	**c**
d	**d**	**a**	a
a	a	d	d

Refugee Preferences P

$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
1	1	4	4
4	3	2	3
2	2	1	1
3	4	3	2

Country Preferences $\succ$

π_a^E	π_b^E	π_c^E	π_d^E
1	**1**	2	1
2	2	1	**2**
4	3	**4**	4
3	4	3	3

Forced Profile π^E

Applying the refugee-proposing DA algorithm to $\succ$ and P gives us the following:

$$\mu^{DA} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ b & d & a & c \end{pmatrix}$$

The matching μ^{DA} satisfies PC fairness because it satisfies fairness. However, matching μ^{DA} does not satisfy PC no-envy. This is because $(1,c)$ and $(2,c)$ are blocking pairs for μ^{DA} with respect to π^E. Refugees 1 and 2 are envious of refugee 4, who is in a lower PC than both the refugees and matched to c in μ^{DA}.

Furthermore, PC no-envy does *not* imply PC fairness. Let μ_E^{DA} be the matching result of the DA when applied to forced hierarchical priority classes. μ_E^{DA} is stable with respect to the forced hierarchical priority classes of π^E at each profile P and hence satisfies PC no-envy. Notably, observe that PC no-envy is satisfied whenever a serial dictatorship procedure is used with the permutation of agents based on the forced hierarchical priority classes. For the purpose of this exercise, suppose an arbitrary common priority ordering of agents π and let f denote a serial dictatorship procedure. Recall that, given an ordering π of agents with any permutation of the entire set of agents, a serial dictatorship $f(\pi)$ (Satterthwaite & Sonnenschein, 1981) assigns the objects to agents as follows: The first agent is assigned their first choice among all the objects. The second agent is assigned their first choice among all the objects, excluding the choice of the first agent, and so on. Now, consider an ordering π of refugees following the fixed hierarchy of priority classes; any ordering is possible within the priority classes as long as it is the same for each country. Then, the PC no-envy property is satisfied whenever $f(\pi)$ is used since $f(\pi)$ assigns the permits to refugees following the given common order. Consider the example below to examine this visually.

Example 2. Consider the problem P below, where refugees $\mathscr{R} = \{1,2,3,4\}$, countries $\mathscr{C} = \{a,b,c,d\}$, and country quotas $q_c = 1$ for all $c \in \mathscr{C}$, and forced priority classes $\mathscr{R}_1 = \{1,2\}$ and $\mathscr{R}_2 = \{3,4\}$. Let π be the permutation. Allocations of the result of the serial dictatorship f using the given common priority order π are in bold.

P_1	P_2	P_3	P_4
a	**a**	**c**	b
b	c	b	c
c	d	a	a
d	b	d	**d**

Refugee Preferences P

π_a	π_b	π_c	π_d
2	2	2	2
1	1	1	1
3	3	3	3
4	4	4	4

Common Priority Order π

π^E_a	π^E_b	π^E_c	π^E_d
1	2	2	1
2	**1**	1	2
4	4	**3**	**4**
3	3	4	3

Forced Profile π^E

The result of the serial dictatorship $f(\pi)$ is as follows:

$$f(\pi) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ b & a & c & d \end{pmatrix}$$

Observe that $f(\pi)$ satisfies PC no-envy as no refugee in a higher forced priority class envies a refugee in a lower priority class. However, it does not satisfy PC fairness because $(1,a)$ is a blocking pair for $f(\pi)$ with respect to π^E. Refugee 1 is envious of 2, who is preferred by a over 2, according to country a's preferences within the top PC of a. Hence, country preferences are violated by $f(\pi)$ within the PC. Therefore, whenever $f(\pi)$ is applied, PC no-envy is satisfied since it is impossible to be envious of a refugee in a lower priority class. However, the result $f(\pi)$ fails PC fairness, as any ordering of refugees is possible within a priority class.

Next, let's formally define the DA that runs taking into account the hierarchical priority classes.

Definition (The DA with Hierarchical Priority Classes). The DA with hierarchical priority classes is the refugee-proposing DA applied to the forced priority profile of countries π^E and refugee preference profile P.

The matching result of the DA with hierarchical priority classes is denoted by μ_E^{DA}.

Proposition 1. Stability with respect to hierarchical priority classes is equivalent to PC no-envy and PC fairness.

Proof. "If" Part. We show that stability with respect to hierarchical priority classes implies PC no-envy and PC fairness. Let μ_E^{DA} be the matching result of the DA with hierarchical priority classes for a given problem P. Given that the DA with hierarchical priority classes is the DA applied to forced priorities π^E, then μ_E^{DA} will be stable with respect to π^E for any given preference profile. Then, there will be no refugee–country pair (r,c) blocking μ_E^{DA} for any given preference profile. This implies two cases.

Case 1. Suppose PC fairness is not satisfied. Then there is a refugee–country pair (r,c) such that r prefers c and is preferred by c for a refugee $\hat{r}$ assigned to c, where r and $\hat{r}$ are in the same forced PC. Hence, there is a ranking violation within a priority class. However, this is a direct contradiction to the stability of the DA procedure with hierarchical priority classes, which is applied with respect to forced priorities π^E.

Case 2. Suppose PC no-envy is not satisfied. Then there is a refugee–country pair (r,c) such that r prefers c and is ranked highly by c, according to c's priority ranking in π^E to a refugee $\hat{r}$ assigned to c, where r and $\hat{r}$ are not in the same forced PC. Then, there is a ranking violation across priority classes.

Specifically, then there is r who is in a higher PC than $\hat{r}$ and has envy for $\hat{r}$. However, this is a direct contradiction to the stability of the DA procedure that is applied using the forced hierarchical priority classes of π^E.

Therefore, the stability of DA with the forced hierarchical priority classes of π^E implies PC no-envy and PC fairness.

"Only If" Part. We show that satisfying PC fairness and PC no-envy implies satisfying stability with respect to hierarchical priority classes.

Case 1. Suppose PC fairness is satisfied. Then, if a refugee–country pair (r,c) is such that r prefers c and is preferred by c for a refugee $\hat{r}$ assigned to c, then $\hat{r}$ must be in a higher PC than r. This implies $\hat{r}$ and r are not in the same PC. Hence, in the same PC, there is no refugee–country pair (r,c) such that r prefers c and is preferred by c for a refugee $\hat{r}$ assigned to c. This satisfies stability within each forced PC.

Case 2. Suppose PC no-envy is satisfied. For each preference profile P, if r is in a higher forced PC than $\hat{r}$, then r does not envy $\hat{r}$. Hence, for any given preference profile P, if $\hat{r} \in \mu_E^{DA}(c)$, then $\mu_E^{DA}(r)\ W_r\ c$. Therefore, there is no refugee–country pair (r,c) such that r prefers c and is ranked above by c according to c's priority ranking in π^E to a refugee $\hat{r}$ assigned to c. This satisfies stability across forced hierarchical priority classes.

Therefore, PC fairness and PC no-envy imply stability with respect to π^E. In conclusion, satisfying the two axioms of PC fairness and PC no-envy is equivalent to satisfying stability with respect to the forced hierarchical priority classes. $\square$

A matching is called *optimal with respect to a stability axiom* if every agent receives at least as good an assignment in this matching as in any other matching satisfying the stability axiom. Specifically, a matching μ is *optimal with respect to PC fairness and PC no-envy* at profile P if, for each refugee $r \in \mathscr{R}$, $\mu(r) = c$ is the most preferred country among all the countries that refugee r could be matched to at any matching satisfying PC fairness and PC no-envy at P, when there is any such country. Given $P \in \mathscr{P}$, a matching mechanism is *optimal with respect to PC fairness and PC no-envy* if, for each preference profile $P \in \mathscr{P}$, it assigns a matching to profile P that is optimal with respect to PC fairness and PC no-envy.

Moreover, the DA with hierarchical priority classes is the DA where countries' preferences are obtained from the ordered priority classes. Correspondingly, within each class, each country c break ties according to $\succ_c$. Hence, the DA with hierarchical priority classes is a unique mechanism that is optimal with respect to PC fairness and PC no-envy.

Theorem 1. A mechanism satisfies PC fairness, PC no-envy, and is optimal with respect to PC fairness and PC no-envy if, and only if, it is the DA with hierarchical priority classes.

Proof. "If" Part. The DA with hierarchical priority classes satisfies PC fairness and PC no-envy. Therefore, this part follows directly from the equivalence between stability with respect to hierarchical priority classes and the combination of PC no-envy and PC fairness.

"Only If" Part. We show that if a mechanism is PC-fair, PC no-envy, and is optimal with respect to PC fairness and PC no-envy, then it is the DA with hierarchical priority classes. First, we know that stability with respect to π^E is equivalent to the combination of PC fairness and PC no-envy. By this equivalence result in Proposition 1, the DA with hierarchical priority classes is optimal with respect to PC fairness and PC no-envy. Focusing on the welfare of refugees, maximized welfare is obtained by the refugee-proposing DA subject to stability with respect to π^E. Hence, this follows from the DA refugee-optimal solution with the priorities π^E. Consequently, since optimality implies uniqueness, we conclude that if a mechanism is PC-fair, PC no-envy, and is optimal with respect to PC fairness and PC no-envy, then it is the DA with hierarchical priority classes. $\square$

It is vital to discuss this result intuitively apropos of the two key weak stability and fairness axioms examined in this study. PC no-envy of any arbitrary matching mechanism implies that a refugee $\hat{r}$ in a higher forced priority class does not have envy of any of the allocations below $\hat{r}$ at any profile P. This implies that the allocation starts at the first PC and follows a serial procedure of priority classes according to a given order of refugees having the fixed hierarchy of priority classes. In this case, any order can be followed within priority classes. We allocate refugees to the top PC, followed by the second PC, and so on, which is equivalent to the serial dictatorship procedure with the permutation of agents based on the hierarchical priority classes. Succeeding the completion of the top priority class allocation, those allocations are finalized and removed. Subsequently, allocations are done for the second priority class refugees, and so on. Instead of individual agents picking their top choices, we have priority classes of refugees who get their top choices, independent of the preferences of refugees in other priority classes.

In addition to PC no-envy, the requirements of PC fairness and optimality for a matching mechanism imply that, for any problem P, the match must be fair within each PC. Thus, the DA must be applied to each PC to ensure fair allocation within each PC. Therefore, since the DA applied to P with a given π^E is the DA with hierarchical priority classes, and π^E is a partition profile, it is equivalent to the DA applied to the top PC than the DA applied to the second PC, and so on, following the given hierarchical priority class ordering. Therefore, if a mechanism satisfies PC fairness, PC no-envy, and is optimal with respect to PC fairness and PC no-envy, then it is the DA with hierarchical priority classes.

5. TOP STABILITY

It is intuitive and vital to consider the top choice countries of refugees when designing a refugee matching system. We recognize that the imposed priority classes force the countries to change their preference rankings, deviating this study from the objective of a fair refugee allocation. Since the imposed priorities give certain refugees a priority in each country, it is crucial to incorporate refugees' preferences into the matching algorithms. A refugee in a lower PC always remains in that PC. Since priority classes are forced in all countries, every refugee in the first PC is always prioritized over every refugee in the second PC. Considering the refugees in the lower priority classes, the proposed model gives these refugees a better chance of matching with their top-ranked country by lifting them up, using their preferences. For interesting and relevant notions, which were studied independently and in different contexts, please refer to Morrill (2013); Pathak & Sönmez (2013); Afacan et al. (2017); and Dur et al. (2018).

For the definitions of weak stability axioms, recall the definition of blocking pairs (r,c) in Section 3. In addition, please note that priority classes that are common to all countries are used in every section. Sections differ only in the newly introduced mechanism designs.

Definition (Top Stability Axiom). A matching is top stable if it cannot be blocked with a pair (r, c), where P_r ranks country c first at a fixed preference profile P. A mechanism is top stable if, for each preference profile P, whenever the matching assigned to P is blocked by (r, c), P_r does not rank c first.

Definition (Top PC Fairness Axiom). A matching μ satisfies top PC fairness if, for every refugee–country (r, c) such that r prefers c at μ and is preferred by c to a refugee $\hat{r}$ assigned to c at μ,

- either $\hat{r}$ is in a higher forced PC than r,
- or $P_{\hat{r}}$ ranks c first, and r is not in PC $\mathscr{R}_1$ of country c.

A mechanism f satisfies top PC fairness if, for every profile P, it assigns a matching μ that is top PC-fair. Notably, whenever there is justified envy with respect to $\hat{r}$, either $\hat{r}$ is in a higher forced class than r with justified envy or $P_{\hat{r}}$ ranks c first, and r is not in the top PC of country c.

To accommodate more refugees in the lower priority classes by raising them to the first PC at their top-ranked country, I further weaken the PC fairness axiom. Therefore, the PC fairness axiom implies top PC fairness. Since I make exceptions for these refugees regarding their preferred country and account for their exceptions of gained rank, I declare blocking pairs involving such refugees and their top choice country to be salient and, therefore, inadmissible. In addition, top stability and top PC fairness axioms are not independent of each other. Connections between top stability and top PC fairness axioms are demonstrated in Example 3.

Moreover, in order to obtain a combined priority ranking profile for countries that merge $\succ$ with π^E, we would need a mechanism that outlines a method to combine the two profiles $\succ$ and π^E. Such a mechanism would ideally be based on refugee preferences P when combining the two profiles $\succ$ and π^E.

Remark. The combined priority profile of countries depends on the refugee preference profile P. However, the forced priority profile of countries is independent of the refugee preference profile P.

One of the two mechanisms designed in this study is the following:

Definition (The Top Prioritization Mechanism).

1. For every refugee r, modify π_c^E by lifting refugee r up to PC $\mathscr{R}_1$ of c, where c is top-ranked in P_r.

2. Position r within PC $\mathscr{R}_1$, according to $\succ_c$ (i.e., keep all the other priority rankings the same, except for r's). This yields the new combined profile π^{E-top}.

3. Apply the DA to the π^{E-top}. This yields the matching result $f_{E-top}^{DA}(P)$ of the top prioritization mechanism f_{E-top}^{DA} at problem P.

Similar to the other notations for the matching outcomes, I use μ_{E-top}^{DA} for the matching result of the top prioritization mechanism f_{E-top}^{DA}, for any given problem P.

Furthermore, given an ordering π_c, let $S_c^{\pi}(r)$ denote the *upper contour set* at r. Then, the upper contour set of refugee r in country c's profile is as follows:

$$S_c^{\pi}(r) = \{\hat{r} \in \mathscr{R} : \hat{r}\ \pi_c\ r\ \}.$$

Lemma 1. If P_r ranks country c first, then $S_c^{\pi^{E-top}}(r) \subseteq S_c^{\succ}(r)$.

Proof. Let r and c be such that P_r ranks country c first. Then, r is lifted to $\mathscr{R}_1$ in π_c^{E-top}. Fix $\hat{r} \in S_c^{\pi^{E-top}}(r)$. Then, we have $\hat{r} \in \mathscr{R}_1$. Moreover, given the construction of π^{E-top}, this means that $\hat{r} \succ_c r$. Thus, $\hat{r} \in S_c^{\succ}(r)$. □

In addition, the top prioritization mechanism guarantees the top stability of μ_{E-top}^{DA} at any P. Thus, we have the following result:

Theorem 2. The top prioritization mechanism is

1. top stable and

2. top PC-fair.

Proof. 1. Top stability: Fix a given preference profile P. For contradiction, suppose the matching result μ_{E-top}^{DA} is not top stable at the given profile P. Then, there is a pair (r,c) blocking μ_{E-top}^{DA} with respect to $\succ$ at P, where refugee r ranks c first at preference profile P. Given that the DA is applied to the combined profile π^{E-top}, if country c rejects r in any round of the DA, then c is temporarily matched to at least one refugee other than r. Let this refugee be $\hat{r}$. Then, this implies $\hat{r}\ \pi_c^{E-top}\ r$. Since r ranks c first at preference profile P, r is lifted up to PC $\mathscr{R}_1$ by country c in π_c^{E-top} by the top prioritization procedure. By Lemma 1, we conclude the following. If r is in the top PC of π_c^{E-top} and $\hat{r}\ \pi_c^{E-top}\ r$, then r and $\hat{r}$ will both be in the top PC. Then, $\hat{r} \succ_c r$ since country preferences are preserved within each PC. This contradicts the assumption that (r,c) is a blocking pair of μ_{E-top}^{DA} with respect to $\succ$. Hence, μ_{E-top}^{DA} is top stable.

2. Top PC-fairness: Fix a given preference profile P. For contradiction, suppose μ_{E-top}^{DA} is not top PC-fair at the given profile P. Then, at the given P, there exists a salient pair (r,c) that blocks μ_{E-top}^{DA} with respect to $\succ$ that is not allowed under top PC fairness. There is $\hat{r} \in \mathscr{R}$ such that $\hat{r} \in \mu_{E-top}^{DA}(c)$. The violation of the top PC fairness of the matching result μ_{E-top}^{DA} at the given P implies four non-trivial cases. We verify the contradictions under each case.

First, observe that when refugee $\hat{r}$ is not in a higher forced PC than r who is assumed to be blocking with c at $\succ$, then they are in the same PC. Although this is trivial, it is a possibility because the matching result is assumed to not be top PC-fair at the given profile. This violates country preferences within the same PC, which are assumed to be preserved under the top prioritization mechanism. Hence, this is a direct contradiction.

Case 1: Suppose $P_{\hat{r}}$ does not rank c first and r is in PC $\mathscr{R}_1$ of country c. Then, either $\hat{r}$ is in a lower PC than r, or $\hat{r}$ is already in $\mathscr{R}_1$. Hence, both r and $\hat{r}$ are in $\mathscr{R}_1$ of country c. If $r \in \mathscr{R}_1$ of c and $\hat{r}$ is in a lower PC, then this contradicts $\hat{r}\ \pi_c^{E-top}\ r$ since $\hat{r} \in \mu_{E-top}^{DA}(c)$. If both $r, \hat{r} \in \mathscr{R}_1$, then it must be that $r \succ_c \hat{r}$, since for contradiction we assumed the existence of a blocking pair (r,c) with respect to $\succ$. Country preferences are preserved within the same priority classes. Hence, $r\ \pi_c^{E-top}\ \hat{r}$. However, we also have $\hat{r}\ \pi_c^{E-top}\ r$ since $\hat{r} \in \mu_{E-top}^{DA}(c)$. Thus, this is a contradiction.

Case 2: Suppose $P_{\hat{r}}$ ranks c first and r is in PC $\mathscr{R}_1$ of country c. Then, $r, \hat{r} \in \mathscr{R}_1$, then $r \succ_c \hat{r}$, which is preserved in $\mathscr{R}_1$ and hence, $r\ \pi_c^{E-top}\ \hat{r}$. However, this contradicts $\hat{r}\ \pi_c^{E-top}\ r$ given $\hat{r} \in \mu_{E-top}^{DA}(c)$.

Case 3: Suppose $P_{\hat{r}}$ does not rank c first and r is not in PC $\mathscr{R}_1$ of country c. If $\hat{r}$ is in forced class $\mathscr{R}_1$, then $\hat{r}\ \pi_c^{E-top}\ r$, and so there is nothing to prove as μ_{E-top}^{DA} is top PC-fair. However, consider otherwise. Then, neither of the refugee families is in $\mathscr{R}_1$. Since we assumed μ_{E-top}^{DA} is not top PC-fair, either

r and $\hat{r}$ are in the same PC or $\hat{r}$ is in a lower PC than r. If they are both in the same PC, then (r,c) blocking μ^{DA}_{E-top} at $\succ$ will imply that $r \succ_c \hat{r}$, which is preserved in the same PC, then $r\ \pi_c^{E-top}\ \hat{r}$. This is a contradiction to $\hat{r}\ \pi_c^{E-top}\ r$, given $\hat{r} \in \mu^{DA}_{E-top}(c)$. If $\hat{r}$ is in a lower PC than r, then r having envy for $\hat{r}$ in a lower PC will be a contradiction of the PC no-envy property of the match. This is because if r is in a higher PC than $\hat{r}$, then r cannot have envy for $\hat{r}$ at preference profile P.

Case 4: Suppose $P_{\hat{r}}$ ranks c first and r is not in PC $\mathscr{R}_1$ of country c. Then, (r,c) is a salient blocking pair that is allowed under top PC fairness. Hence, this is a direct contradiction.

Therefore, the top prioritization mechanism is top stable and top PC-fair.

□

The next example demonstrates how we achieve the combination of the forced priority classes and the preference rankings of countries, and how we further modify the resulting country rankings by using the top prioritization mechanism to provide higher rank for refugees with their top-ranked countries. The example also verifies that the resulting matching satisfies the weakened axioms of top stability and top PC fairness.

Example 3. Consider the given refugee families $\mathscr{R} = \{1,2,3,4,5,6\}$, countries $\mathscr{C} = \{a,b,c,d\}$, quota vector $q = (1,1,2,2)$, and forced priority classes $\mathscr{R}_1 = \{1,2\}$ and $\mathscr{R}_2 = \{3,4,5,6\}$. Since countries c and d have two residency quotas each, let there be two identical copies of country c, each with preferences $\succ_c$ and capacity one. Similarly, let there be two identical copies of d each with $\succ_d$ and capacity one. The underlined allocations are for the matching result μ^{DA}_{E-top} of the top prioritization mechanism.

P_1	P_2	P_3	P_4	P_5	P_6
d	d	$\underline{d}$	$\underline{d}$	a	a
$\underline{a}$	$\underline{b}$	c	c	d	d
b	c	a	a	$\underline{c}$	$\underline{c}$
c	a	b	b	b	b

Refugee Preferences P

$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
$\underline{1}$	$\underline{2}$	3	5
5	1	4	6
6	5	$\underline{5}$	$\underline{3}$
2	6	$\underline{6}$	$\underline{4}$
3	3	2	1
4	4	1	2

Country Preferences $\succ$

π_a^E	π_b^E	π_c^E	π_d^E
1	2	2	1
2	1	1	2
5	5	3	5
6	6	4	6
3	3	5	3
4	4	6	4

Forced Profile π^E

π_a^{E-top}	π_b^{E-top}	π_c^{E-top}	π_d^{E-top}
1	2	2	3
5	1	1	4
6	5	3	1
2	6	4	2
3	3	5	5
4	4	6	6

Combined Profile π^{E-top}

Observe that P_1, P_2, P_3, and P_4 rank country d first, and P_5 and P_6 rank country a first. Notably, countries b and c are not popular enough among refugees to be top-ranked. Thus, the following countries lift to their top PC $\mathscr{R}_1$ those refugees that top-ranked them: country d lifts up $\{1,2,3,4\}$ and country a lifts up 5 and 6. This gives us the combined priority profile π^{E-top} above.

After applying the DA to π^{E-top} and P, we obtain the matching below:

$$\mu_{E-top}^{DA} = \begin{pmatrix} a & b & c & d \\ 1 & 2 & \{5,6\} & \{3,4\} \end{pmatrix}$$

Observe that the blocking pair set of μ_{E-top}^{DA} with respect to $\succ$ comprises only $(5,d)$ and $(6,d)$. Since d is not the top choice of either of the refugees 5 and 6, the top stability of μ_{E-top}^{DA} is satisfied for the given problem P. Moreover, the reason for refugees 5 and 6 blocking μ_{E-top}^{DA} with country d with respect to $\succ$ is the loss of country d to refugees 3 and 4. It is also attributed to the priority reversal between $\{5,6\}$ and $\{3,4\}$, which stems from P_3 and P_4 ranking d as their top choice, and this leads to $\{3,4\}$ getting lifted to the top priority class of country d and gaining a rank over refugees 5 and 6. Therefore, top PC fairness of μ_{E-top}^{DA} is satisfied.

In this example, we can also see how top stability can imply top PC fairness. In the combined profile, π^{E-top} refugees are already moved up to the top PC of their top choice countries. Refugees $\{5,6\}$ have justified envy of $\{3,4\}$ since $\{3,4\}$ are moved up to the top PC of d. The blocking pairs $(5,d)$ and $(6,d)$ are therefore allowed under top PC fairness.

A mechanism f is *strategy-proof* if for all $r \in \mathscr{R}$, all $P \in \mathscr{P}$, and all P'_r, $f_r(P)\ W_r\ f_r(P'_r, P_{-r})$. A coalition $L \subseteq \mathscr{R}$ can *manipulate* matching $f(P)$ at P

if there exists P'_L such that for all $r \in L$, $f_r(P'_L, P_{-L})\ P_r\ f_r(P)$.[4]

Remark. The top prioritization mechanism is not strategy-proof for refugee families. Now we will prove this statement. Let P be the original refugee preference profile and P' be the misreported refugee preference profile. Let $P_4 = aP_4dP_4cP_4b$ such that the only difference between these two profiles is that refugee 4 misreports its own top choice country as d instead of the truthful a. For simplicity, suppose $q_c = 1$ for each $c \in \mathscr{C}$. It must be noted that refugee $f^{DA}_{E-top}(P)(4) = c$, where $f^{DA}_{E-top}(P)(4)$ is the outcome assigned to refugee 4 by the top prioritization mechanism f^{DA}_{E-top} at P. Regard the given priority classes $\mathscr{R}_1 = \{1,2\}$ and $\mathscr{R}_2 = \{3,4\}$.

P_1	P_2	P_3	P'_4
d	d	d	d
a	b	c	a
b	c	a	c
c	a	b	b

Refugee Preferences P'

$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
1	2	3	4
4	1	4	3
2	4	2	1
3	3	1	2

Country Preferences $\succ$

π^E_a	π^E_b	π^E_c	π^E_d
1	2	2	1
2	1	1	2
4	4	3	4
3	3	4	3

Forced Profile π^E

$\pi^{E-top'}_a$	$\pi^{E-top'}_b$	$\pi^{E-top'}_c$	$\pi^{E-top'}_d$
1	2	2	4
2	1	1	3
4	4	3	1
3	3	4	2

Combined Priority $\pi^{E-top'}$

After applying the DA to $\pi^{E-top'}$ and P', we have the matching below:

$$\mu^{DA'}_{E-top} = \begin{pmatrix} a & b & c & d \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

Therefore, refugee 4 manipulates the top prioritization mechanism f^{DA}_{E-top} at P since there exists P'_4 such that $f^{DA}_{E-top}(P'_4, P_{-4})(4)\ P_4\ f^{DA}_{E-top}(P)(4)$. Hence, refugee 4 manipulates the mechanism by getting their untruthful top choice.

[4] Alternative notations to $f_r(P'_L, P_{-L})$ and $f_r(P)$ are $f(P'_L, P_{-L})(r)$ and $f(P)(r)$, respectively.

6. CREDIBLE STABILITY

Turning to the refugees forced to be in the lower priority classes, I recognize the need for and importance of exploring the means of giving these refugees an additional chance of attaining a higher rank. As mentioned, I explored this by considering two different forms of priority profiles. The first form is obtained by the top prioritization mechanism, which gives these refugees a better chance of matching with their top-ranked country. In this section, I describe my second form, under which I provide these refugees a better opportunity of accessing their DA-matched country. For a relevant mechanism studied independently and in a school choice context, please see Biró & Gudmundsson (2021). In their study, they investigate the complexity of finding Pareto-efficient allocations of highest welfare by providing top priority to all students at the schools where they are assigned in the socially optimal solution.

Unlike the top prioritization mechanism, I find that the DA matching is the best for refugees in a stable matching, which shows the importance of prioritizing these refugees in their DA-matched countries. This prioritization would also be more compelling for countries because *each* country's priorities must undergo *fewer* quota-based modifications than the top prioritization mechanism. This is because, under the top prioritization mechanism, it will not be necessary for some countries to modify their priority orders as much as the popular countries. For example, less-preferred countries for settlement, such as Poland, are less likely to be top-ranked by refugees. These countries will have fewer refugees requiring a promotion to the top PC. However, popular countries, such as Germany, may have to modify their priority rankings to a greater extent. If all refugees rank Germany as their top country of choice, then all refugees will be moved up to the top PC of Germany. Within a class, Germany will rank according to its own preferences based on its points system. If a country is popular, it will be required to make several modifications to its priority ordering, thereby deviating more from the forced priority classes.

I start by defining a *credible blocking pair* to build the new weak stability axiom. Here, I would like to remind the readers that the matching result of the DA ran with $\succ$, and P is denoted by μ^{DA}.

Definition (Credible Blocking Pair). A pair blocking a matching μ is credible if it is matched under μ^{DA}.

A pair blocking a matching μ is *non-credible* if it is *not* matched under μ^{DA}.

Definition (Credible Stability Axiom). A matching is credibly stable if it cannot be blocked with a credible blocking pair at a given P. A mechanism is credibly stable if, for each preference profile P, whenever the matching assigned to P is blocked by (r,c), (r,c) is not a credible blocking pair at P.

Definition (Credible PC Fairness Axiom). A matching μ satisfies credible PC fairness at a particular profile P if, for every refugee–country pair (r,c) such that r prefers c at μ and is preferred by c to a refugee $\hat{r}$ assigned to c at μ,

- either $\hat{r}$ is in a higher forced PC than r,
- or $\hat{r} \in \mu^{DA}(c)$ at P and r is not in PC $\mathscr{R}_1$ of country c.

A mechanism f satisfies credible PC fairness if, for every profile P, it assigns a matching μ that is credibly PC-fair. Notably, whenever there is justified envy with respect to $\hat{r}$, either $\hat{r}$ is in a higher forced class than r with justified envy or $\hat{r} \in \mu^{DA}(c)$ at P and r is not in top PC of country c. To accommodate more refugees in lower priority classes by raising them to the first priority class at their DA-matched country, I further weaken the PC fairness axiom. Therefore, the PC fairness axiom implies credible PC fairness. Since I make exceptions for these refugees with their DA-matched country and account for their exceptions of gained rank, I declare blocking pairs that involve such refugees and their DA-matched country to be salient and inadmissible. In addition, these two axioms are not independent of each other. Connections between credible stability and credible PC fairness axioms are demonstrated as part of Example 4.

Definition (The DA-Match Prioritization Mechanism).

1. For every refugee r, modify π_c^E by lifting refugee r up to PC $\mathscr{R}_1$ of c, where $r \in \mu^{DA}(c)$.
2. Position r within PC $\mathscr{R}_1$ according to $\succ_c$ (i.e., keep all the other priority rankings the same, except for r's). This yields the new combined profile π^{E-DA}.

3. Apply the DA to π^{E-DA}. This yields the matching result $f^{DA}_{E-DA}(P)$ of the DA-match prioritization mechanism f^{DA}_{E-DA} at problem P.

Similar to the other aforementioned notations for the matching outcomes, I use μ^{DA}_{E-DA} for the matching result of the DA-match prioritization mechanism f^{DA}_{E-DA} for any given problem P.

Unlike the top prioritization mechanism, each country's DA-matched refugees get lifted to the PC $\mathscr{R}_1$. Under the top prioritization mechanism, only the top-ranked countries lift refugees up to their PC $\mathscr{R}_1$. However, I now lift the DA-matched refugees to their DA-matched country's PC $\mathscr{R}_1$. The key distinction is that the DA-match is used to promote the refugees, which is an assignment, unlike the top choices. Thus, each country gets to lift its DA-matched refugees.

Lemma 2. If $r \in \mu^{DA}(c)$, then $S_c^{\pi^{E-DA}}(r) \subseteq S_c^{\succ}(r)$.

Proof. Let r and c be such that $r \in \mu^{DA}(c)$. Then, r is lifted to $\mathscr{R}_1$ in π_c^{E-DA}. Fix $\hat{r} \in S_c^{\pi^{E-DA}}(r)$. Then, we have $\hat{r} \in \mathscr{R}_1$. Moreover, given the construction of π^{E-DA}, this means that $\hat{r} \succ_c r$. Thus, $\hat{r} \in S_c^{\succ}(r)$. $\square$

Lemma 3. μ^{DA} is stable with respect to the priority profile π^{E-DA}.

Proof. For contradiction, suppose there is a pair (r,c) blocking μ^{DA} with respect to π^{E-DA}. Let r be one of the $|\mu^{DA}(\bar{c})|$ refugees matched to $\bar{c}$. In other words, let $r \in \mu^{DA}(\bar{c})$. Then, $c \; P_r \; \bar{c}$. Let $\hat{r}$ be one of the $|\mu^{DA}(c)|$ refugees matched to c. In other words, let $\hat{r} \in \mu^{DA}(c)$. Then, $r \; \pi_c^{E-DA} \; \hat{r}$. Thus, $r \in S_c^{\pi^{E-DA}}(\hat{r})$, and Lemma 2 implies that $r \in S_c^{\succ}(\hat{r})$. Then (r,c) is also blocking μ^{DA} with respect to $\succ$, which is a contradiction, since μ^{DA} is stable at $\succ$. $\square$

Lemma 3 can occur in two cases: μ^{DA} is stable with respect to π^{E-DA} either by simply being equal to μ^{DA}_{E-DA} or when these two matches differ. Through the DA-match prioritization mechanism, we have $\mu^{DA}_{E-DA} = \mu^{DA}$ implying, μ^{DA} is still stable with respect to π^{E-DA}, and even μ^{DA}_{E-DA} is stable with respect to $\succ$. We can also have $\mu^{DA}_{E-DA} \neq \mu^{DA}$, where μ^{DA} is still stable with respect to π^{E-DA}, and μ^{DA}_{E-DA} Pareto dominates μ^{DA} for refugees, which I prove in the next theorem.

Remark. When μ^{DA}_{E-DA} coincides with μ^{DA}, then both μ^{DA} and μ^{DA}_{E-DA} are stable with respect to both $\succ$ and π^{E-DA}.

Definition (Weak Pareto Domination). A mechanism f weakly Pareto dominates another mechanism g if, for every P, the matching assigned to f weakly Pareto dominates the matching assigned to g.

Theorem 3. The DA-match prioritization mechanism weakly Pareto dominates the DA for the refugees.

Whenever the matching results do not coincide at a particular profile P, then the DA-match prioritization mechanism leads to a matching μ^{DA}_{E-DA} that Pareto dominates μ^{DA} for refugees.

Proof. From Gale & Shapley (1962)'s optimality result, we know that the refugee-proposing DA-match prioritization leads to the refugee-optimal stable matching with respect to π^{E-DA}. Since μ^{DA}_{E-DA} is the refugee-optimal stable matching at π^{E-DA} and matching μ^{DA} is also stable with respect to π^{E-DA} by Lemma 3, if $\mu^{DA}_{E-DA} \neq \mu^{DA}$, then μ^{DA}_{E-DA} Pareto dominates μ^{DA} with respect to P. □

Furthermore, according to Knuth (1976)'s polarity result, for every problem with strict preferences, both sides of the market have common preferences and these common preferences are opposed to each other on the set of stable matchings. We know that the refugees' preferences P and the combined profile for the countries π^{E-DA} are opposed to each other on the set of stable matchings. For example, following Knuth's polarity, consider μ and μ' as two stable matchings. Then, all the refugees like μ at least as well as μ' if and only if all countries like μ' at least as well as μ. Intuitively, the best stable matching for one side is the worst stable matching for the other side. Thus, the refugee-optimal stable matching is the worst stable matching for countries (country-pessimal), and the country-optimal stable matching is the worst stable matching for refugees (refugee-pessimal). Therefore, as per Knuth (1976) polarity result and by Lemma 3, I observe that when μ^{DA}_{E-DA} differs from μ^{DA}, we have the matching μ^{DA}_{E-DA}—the country-pessimal stable matching with respect to π^{E-DA}. Thus, from the viewpoint of countries, μ^{DA}, which is stable at π^{E-DA}, Pareto dominates μ^{DA}_{E-DA} at π^{E-DA}.

Moreover, the DA-match prioritization mechanism leads to a matching μ^{DA}_{E-DA} that satisfies credible stability. Recall that a credible blocking pair is a pair that is matched under μ^{DA}.

Theorem 4. The DA-match prioritization Mechanism is

1. credibly stable and

2. credibly PC fair.

Proof. 1. Credible stability: Fix a given preference profile P. Suppose $\mu^{DA}_{E-DA} \neq \mu^{DA}$. For contradiction, suppose the matching result μ^{DA}_{E-DA} is not credibly stable at the given profile P. Then, there is a pair (r,c) blocking μ^{DA}_{E-DA} with respect to $\succ$ that is matched under μ^{DA}. Let r be one of the $|\mu^{DA}_{E-DA}(\bar{c})|$ refugees matched to $\bar{c}$ under the DA-match prioritization mechanism. In other words, $r \in \mu^{DA}_{E-DA}(\bar{c})$. We know $\mu^{DA}_{E-DA} \neq \mu^{DA}$ and $c \neq \bar{c}$. Thus, by Theorem 3, $\bar{c}\ P_r\ c$, this contradicts the assumption that (r,c) is a blocking pair of the matching μ^{DA}_{E-DA} with respect to $\succ$, given the profile P.

2. Credible PC fairness: Fix a given preference profile P. For contradiction, suppose μ^{DA}_{E-DA} does not satisfy the credible PC fairness at P. Then, at the given P, there exists a pair (r,c) blocking μ^{DA}_{E-DA} with respect to $\succ$ that is not allowed under credible PC fairness and there exists $\hat{r} \in \mathscr{R}$ such that $\hat{r} \in \mu^{DA}_{E-DA}(c)$. The violation of the credible PC fairness of the matching result μ^{DA}_{E-DA} at the given P implies four non-trivial cases. We verify the contradiction in each case.

First, observe that when refugee $\hat{r}$ is not in a higher forced PC than r, who is assumed to be blocking with c at $\succ$, then they are in the same PC. Although this is trivial, it is a possibility since the matching result is assumed to be not credible PC-fair at the given profile. This violates the country preferences within the same PC, which are assumed to be preserved under the DA-match prioritization mechanism. Hence, this is a direct contradiction.

Case 1: Suppose $\hat{r} \notin \mu^{DA}(c)$ at given P, and r is in top PC $\mathscr{R}_1$ of country c. Then, either $\hat{r}$ is in a lower PC than r or $\hat{r}$ is already in top PC $\mathscr{R}_1$, and hence they are both in $\mathscr{R}_1$ of country c. If $r \in \mathscr{R}_1$ of c and $\hat{r}$ is in a lower PC, then this will contradict $\hat{r}\ \pi^{E-DA}_c\ r$ since $\hat{r} \in \mu^{DA}_{E-DA}(c)$. If both $r, \hat{r} \in \mathscr{R}_1$, then $r \succ_c \hat{r}$, and thus it should be preserved in $\mathscr{R}_1$. Then, $r\ \pi^{E-DA}_c\ \hat{r}$. However, this contradicts $\hat{r}\ \pi^{E-DA}_c\ r$, given $\hat{r} \in \mu^{DA}_{E-DA}(c)$.

Case 2: Suppose $\hat{r} \in \mu^{DA}(c)$ at P, and r is in PC $\mathscr{R}_1$ of country c. Then, $r, \hat{r} \in \mathscr{R}_1$ and by our assumption we have $r \succ_c \hat{r}$, which is preserved in $\mathscr{R}_1$ and so $r \; \pi_c^{E-DA} \; \hat{r}$. However, given $\hat{r} \in \mu_{E-DA}^{DA}(c)$ we also have $\hat{r} \; \pi_c^{E-DA} \; r$, which gives us a contradiction.

Case 3: Suppose $\hat{r} \notin \mu^{DA}(c)$ at P, and r is not in PC $\mathscr{R}_1$ of country c. If $\hat{r}$ is already in forced class $\mathscr{R}_1$, then $\hat{r} \; \pi_c^{E-DA} \; r$, and thus there is nothing to prove as μ_{E-DA}^{DA} is credibly PC-fair. However, considering otherwise, neither of the refugee families is in $\mathscr{R}_1$. Hence, since we assumed μ_{E-DA}^{DA} is not credible PC-fair, either r and $\hat{r}$ are in the same priority class, or $\hat{r}$ is in a lower PC than r. If they are both in the same PC, then (r, c) blocking μ_{E-DA}^{DA} at $\succ$ implies that $r \succ_c \hat{r}$, which is preserved in the same PC. Hence, $r \; \pi_c^{E-DA} \; \hat{r}$. This is a contradiction to $\hat{r} \; \pi_c^{E-DA} \; r$, given $\hat{r} \in \mu_{E-DA}^{DA}(c)$. If $\hat{r}$ is in a lower PC than r, then r having envy for $\hat{r}$ who is in a lower PC, is a contradiction to the PC no-envy property of the match. This is because if r is in a higher PC than $\hat{r}$, then r does not have envy for $\hat{r}$ at P.

Case 4: $\hat{r} \in \mu^{DA}(c)$ at P and r is not in PC $\mathscr{R}_1$ of country c. Then, (r, c) is a salient blocking pair allowed under credible PC fairness. Hence, this is a direct contradiction.

Therefore, the DA-match prioritization mechanism satisfies credible stability and credible PC fairness. □

The next example demonstrates the DA-match prioritization mechanism. It shows how we make exceptions for refugees by moving them up to the top priority class of their DA-matched countries. We also observe how the DA-match prioritization mechanism differs from the top prioritization mechanism. In contrast to the top prioritization mechanism, as seen in Example 3, we first need to obtain the DA matching result using refugee and country preferences before adjusting the country rankings accordingly. The example also shows how to verify that the weakened axioms of credible stability and credible PC fairness hold for the matching obtained using the DA-match prioritization mechanism. Finally, it demonstrates how the result of the DA-match prioritization mechanism can Pareto dominate the DA outcome for refugees.

Example 4. Consider the given refugees $\mathscr{R} = \{1,2,3,4,5,6,7,8\}$, countries $\mathscr{C} = \{a,b,c,d\}$, and forced priority classes $\mathscr{R}_1 = \{1,2,5,6\}$ and $\mathscr{R}_2 = \{3,4,7,8\}$. This example demonstrates the case where the DA-match prioritization mechanism leads to a matching μ_{E-DA}^{DA} that does not coincide with μ^{DA}.

Suppose $q = (2,2,2,2)$. The double-underlined allocations are for μ^{DA}_{E-DA}, which is obtained from applying the DA to the P and π^{E-DA}. The underlined allocations are for μ^{DA}. Allocations that are marked in bold show the instance when μ^{DA}_{E-DA} coincides with μ^{DA}.

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
$\underline{\underline{\text{d}}}$	d	d	$\underline{\underline{\text{a}}}$	d	d	d	a
$\underline{\text{a}}$	**b**	**c**	$\underline{\text{d}}$	**a**	**b**	**c**	**d**
b	c	a	c	b	c	a	c
c	a	b	b	c	a	b	b

Refugee Preferences P

$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
$\underline{1}$	**2**	**3**	$\underline{4}$
5	**6**	**7**	**8**
$\underline{\underline{4}}$	1	2	3
8	5	6	7
2	3	1	$\underline{\underline{1}}$
6	7	5	2
3	4	4	5
7	8	8	6

Country Preferences $\succ$

π^E_a	π^E_b	π^E_c	π^E_d
1	2	2	1
5	6	6	2
2	1	1	5
6	5	5	6
4	3	3	4
8	7	7	8
3	4	4	3
7	8	8	7

Forced Profile π^E

π^{E-DA}_a	π^{E-DA}_b	π^{E-DA}_c	π^{E-DA}_d
1	2	3	4
5	6	7	8
2	1	2	1
6	5	6	2
4	3	1	5
8	7	5	6
3	4	4	3
7	8	8	7

Combined Profile π^{E-DA}

After applying the DA to π^{E-DA} and P, we get:

$$\mu^{DA}_{E-DA} = \begin{pmatrix} a & b & c & d \\ \{4,5\} & \{2,6\} & \{3,7\} & \{1,8\} \end{pmatrix}$$

$$\mu^{DA} = \begin{pmatrix} a & b & c & d \\ \{1,5\} & \{2,6\} & \{3,7\} & \{4,8\} \end{pmatrix}$$

Applying the DA to P and π^{E-DA} gives the refugee-optimal stable matching μ^{DA}_{E-DA} at π^{E-DA}. The two matching results above are different from each other. Looking at P, observe that μ^{DA}_{E-DA} Pareto dominates μ^{DA} for refugees. At P, refugees 1 and 4 are better off under the matching μ^{DA}_{E-DA}. It must be noted that $(3,d)$ and $(7,d)$ are the only pairs blocking μ^{DA}_{E-DA} with respect to $\succ$. Since $(3,d)$ and $(7,d)$ are not matched in μ^{DA}, they are not credible. Therefore, these blocking pairs are allowed to exist under the weak stability of credible stability. Thus, the matching μ^{DA}_{E-DA} is credibly stable. In addition, the reason for refugees 3 and 7 blocking μ^{DA}_{E-DA} with respect to $\succ$ is the loss of country d to refugee 1 owing to the PC reversals between 1 and 3 and between 1 and 7. These priority reversals stem from the discrepancy between π^E and $\succ$. In other words, 1 is in a higher forced PC than 3 and 7 in π^E. This leads to the following reversal: $3 \succ_d 1$ versus $1\ \pi^E_d\ 3$ and $7 \succ_d 1$ versus $1\ \pi^E_d\ 7$. Therefore, μ^{DA}_{E-DA} *is credibly PC-fair because it is PC-fair.*

This example also shows how credible stability can imply credible PC fairness. Notably, none of the blocking pairs $(3,d)$ and $(7,d)$ are matched under the DA. In the combined profile π^{E-DA}, refugees are already lifted to the top priority class of their DA-matched countries. Therefore, the priority reversal between refugees $\{3,7\}$ and 1 is attributed to the forced priority classes—the difference between the forced profile of countries π^E and country preferences $\succ$. Refugees 3 and 7 have justified envy for 1 because 1 is forced above these other two refugees in π^E. Subsequently, these blocking pairs are allowed under PC fairness, and PC fairness implies credible PC fairness.

Remark. The DA-match prioritization mechanism is not strategy-proof for refugee families.

Proof. The matching assigned by the DA-match prioritization mechanism Pareto dominates the matching made by the DA at each preference profile. Thus, whether by Kesten (2006) and Ergin (2002), or Kesten & Kurino (2019), the DA-match prioritization mechanism cannot be strategy-proof.[5] $\square$

[5] Kesten (2006) or Kesten & Kurino (2019) are more relevant here, given that there are no outside options for refugees in the present setting. Also note that this holds per Abdulkadiroğlu et al. (2009) too, as well as by Alva & Manjunath (2019), which require refugees to have an outside option.

7. CONCLUSION

To coordinate the stream of refugees effectively, it would be ideal to have as many European countries as possible participating in a centralized matching mechanism. Persuading all the European countries to participate in a centralized computerized matching mechanism has its challenges. This study hopes to contribute to the efforts to overcome the political impasse that tends to prevent European countries from participating in accountability-sharing in the international refugee crisis. By defining and investigating a country acceptance problem, this study proposes two matching mechanisms. These mechanisms are based on the proposed priority class hierarchy using the UNHCR humanitarian principles and guidelines. They contribute to the literature by offering methodologies that reconcile country preferences and the UNHCR-mandated hierarchical priority classes while maintaining the laudable stability and fairness properties.

Having two kinds of ranking profiles for countries, the UNHCR-mandated priority profile and the preference profile, allows for an examination of the real-world predicaments associated with the refugee reallocation problem. I capture how the difference between the two ranking profiles creates blocking pairs of countries and refugees owing to the forced hierarchical priority classes. I weaken the stability axiom since a mechanism that is stable with respect to a forced profile may no longer be stable with respect to countries' preferences. This may lead the country and refugee pairs to block a matching result. With regard to the top prioritization mechanism, I provide an additional chance to refugees forced in lower priority classes and who therefore face the risk of remaining in the lower priority classes. To this end, I prioritize these refugees in their top choice countries. The DA-match prioritization mechanism allows lifting refugees forced in lower priority classes to the first PC of their DA-matched country. I find that, for refugees, the DA-match prioritization mechanism weakly Pareto dominates the DA mechanism. Furthermore, I recognize the importance of persuading countries to participate in a centralized refugee matching mechanism. I believe that a priority class-based approach with no imposed category-specific set-aside reserve quotas may be more attractive in terms of increasing countries' willingness and incentive to participate in solving the refugee matching problem. Beyond the refugee reallocation context, this study's results have other policy applications, such as centralized college admissions, the design of public-school choice systems, and forced migration or displacement.

This study's results may also apply in a school choice or college admission context, more so than in the context of refugee settlement. This criticism may arise because refugee allocation has some features that do not necessarily fit into the model presented. Four such factors are as follows: (i) the preferences of the refugees may not be considered or even if considered, the optimization may be focused on other factors; (ii) the size of the families can matter, as the quotas are typically for a specific number of people admitted, and further constraints may also be implied for other characteristics of the families; (iii) refugee allocation has a dynamic nature; and (iv) the usage of stability for refugee allocation in Europe may cause skepticism for political reasons, as it could cause unbalanced solutions, with the most attractive countries receiving the "best" (highly qualified, easy to settle) refugees.

With my study, I endeavour to contribute to the literature at the intersection of matching theory and refugee studies, which are less extensive than the school choice literature. This study does not seek to address all aspects of refugee matching. Addressing factors such as family size, the dynamic aspect, and finding a more balanced (fair) solution all at once can be significantly challenging. Instead, this study aims to introduce a new aspect to the choice literature, that is, hierarchical classes, which is at the proposed model's core. By emphasizing the PC hierarchy, this study highlights different theoretical aspects. Moreover, country preferences are frowned upon in refugee settlement due to discrimination issues. Therefore, hierarchical classes are more critical in the refugee settlement context as they significantly curb potential discrimination by imposing the UNHCR-based priority classes. This, therefore, makes country preferences much less important for the matching outcome.

Further, as the Syrian war prompted me to conduct this study, I focus on the static matching problem and earmark dynamic management for future work, in which I will extend my model with hierarchical classes to a dynamic setting. I will also consider investigating the incorporation of hierarchical classes in dynamic capacity management. In addition, future work should consider improving refugee allocation and integration through data-driven algorithmic assignments with hierarchical classes.

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